Student Solution Manual for Deterministic Operations Research

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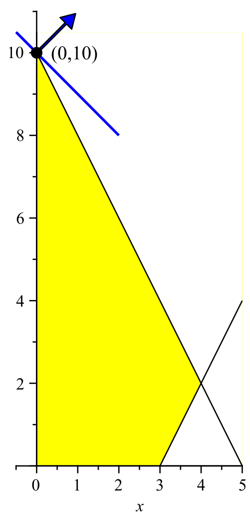
Solutions to selected problems are included in this file. The problems selected are primarily computational in nature; the more theoretical questions are left for you to answer.

Check back often for any updates to this file, as more problems may be added in the future.

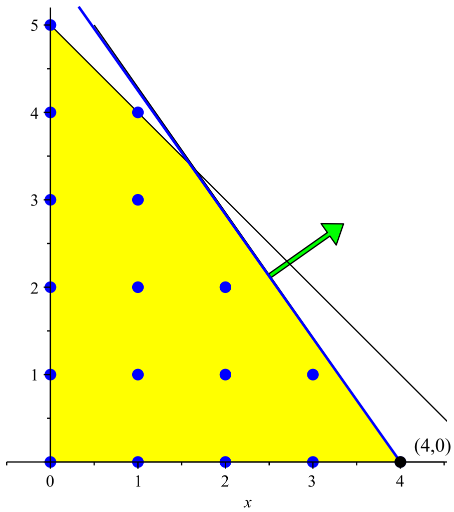
Chapter 1

1.1 (a) The optimal solution occurs at $(0, 10)$ with optimal value 20.



1.2 (a) The optimal range for coefficient c_x of x is $c_x \leq 4$. The range for coefficient c_y of y is $c_y >= 1.5$.

1.3 The optimal solution occurs at $(4, 0)$ with optimal value 68.


5
4
3
2
1
0
1
2
3
4
x
(4,0)

Chapter 2

Note: Many of these solutions are available electronically in a variety of modeling languages. Many of the solutions given here are presented in the modeling language Mosel (Xpress-MP) as an example. In addition, we often provide a general model (without the data values) as opposed to a specific instance.

2.1 Let UT, UC denote the number of unfinished tables and chairs made, FT, FC be the number of finished tables and chairs made, and W be the amount of wood purchased. Our model is then

$$
\begin{aligned}
\max \quad & 80UT + 120FT + 30UC + 55FC - 2W \\
\text{s.t.} \quad & \\
& 4UT + 2UC + 12FT + 8FC \leq 2500 && \text{(Hours)} \\
& W = 25(UT + FT) + 10(UC + FC) && \text{(Wood)} \\
& W \leq 10000 \\
& UC + FC \geq 2(UT + FT) \\
& UC + FC \geq 450 \\
& UT + FT \geq 200 \\
& UT, UC, FT, FT, W \geq 0.
\end{aligned}
$$

The optimal solution is $UT = 130$, $FT = 90$, $UC = 450$, $FC = 0$, and $W = 10000$ with value 14700.

2.2 Below summarizes the changes in optimal solution and profit for each amount of labor.

Labor Hours	Solution	Profit
290	$(9.35484, 0, 9.35484)$	935.484
300	$(9.67742, 0, 9.67742)$	967.742
310	$(10, 0, 10)$	1000
320	$(5.92593, 3.7037, 9.62963)$	1000

As labor increases to 310 hours, we do not make any High Security doors in the optimal profit because their labor usage is too high. Once we have enough labor, we begin to produce these doors.

2.4 Let A, B, C denote the number of stained bookshelves and UA, UB, UC the number of unstained bookshelves to produce. Our model is as follows.

$$
\begin{array}{lll}
\max & 60A + 40B + 75C + 30UA + 20UB + 40UC & \\
\text{s.t.} & & \\
& (A + UA) + 0.5(B + UB) + 2(C + UC) \leq 200 & \text{(cutting)} \\
& 4(A + UA) + 3(B + UB) + 6(C + UC) \leq 700 & \text{(assembly)} \\
& 7A + 5B + 8C \leq 550 & \text{(staining)} \\
& UA + UB + UC \leq 50 & \text{(Unstained Max)} \\
& B + UB \geq 20 & \\
& \quad A, B, C, UA, UB, UC \geq 0 &
\end{array}
$$

(a) The optimal solution is to $A = 30$, $B = 20$,$C = 30$, and $UC = 50$, for a profit of \$6850.

(b) There are only 40 hours available in Assembly. The others use all available hours.

(c) When the profit margin is 50 or 55, the optimal solution changes to $A = 0$, $B = 50$,$C = 37.5$, and $UC = 50$. When the profit margin is 65, the optimal solution remains at $A = 30$, $B = 20$,$C = 30$, and $UC = 50$. When the profit margin is 70, the solution become $A = 64.2857$, $B = 20$,$C = 0$, and $UC = 50$

(d) When the profit margin is 20 and 25, the solution remains as it was for 30: $A = 30$, $B = 20$,$C = 30$, and $UC = 50$. When the profit margin becomes 35, the solution changes to $A = 0$, $B = 20$,$C = 56.25$, $UA = 22.5$, and $UC = 27.5$. When the profit margin is 40, the solution becomes $A = 0$, $B = 20$,$C = 56.25$, and $UA = 50$.

(e) When cutting hours are 150, the optimal solution is $A = 64.2857$, $B = 20$,$C = 0$, $UA = 24.2857$, and $UC = 27.7143$ for a profit of \$6414.29. When cutting hours are 175, the optimal solution is $A = 63.3333$, $B = 20$,$C = 0.83333$, $UA = 0$, and $UC = 50$ for a profit of \$6662.50. When cutting hours are 225 and 250, the optimal solution is $A = 0$, $B = 20$,$C = 56.25$, $UA = 0$, and $UC = 50$ for a profit of \$7018.75.

(f) When assembly hours are 650, the optimal solution is $A = 38$, $B = 20$,$C = 23$, and $UC = 50$, for a profit of \$6805. When assembly hours are 675, 700, 725, and 750, the optimal solution is $A = 30$, $B = 20$,$C = 30$, and $UC = 50$, for a profit of \$6850.

2.6 If we let $Emp8hr(j)$ and $Emp12hr(j)$ denote the number of 8-hour and 12-hour workers starting their shift in time slot j, then our model (in Mosel) would be as follows.

```
forall(i in 1..2) do
  PERIOD12(i):= sum(j in maxlist(i-2, 1)..i) Emp8hr(j) +
                sum(j in minlist(i+5, 7)..6) Emp8hr(j) +
                Emp12hr(1)
                >= minpeople(i)

  Emp8hr(i) is_integer
  Emp12hr(i) is_integer
end-do

PERIOD3 := sum(j in 2..3) Emp8hr(j) +Emp12hr(1) + Emp12hr(3) >= minpeople(3)
```

```
Emp8hr(3) is_integer
Emp12hr(3) is_integer

forall(i in 4..5) do
  PERIOD45(i):= sum(j in maxlist(i-2, 1)..i) Emp8hr(j) + Emp12hr(3) >= minpeople(i)

  Emp8hr(i) is_integer
  Emp12hr(i) is_integer
end-do

PERIOD6 := sum(j in 5..6) Emp8hr(j) >= minpeople(6)

Emp8hr(6) is_integer
Emp12hr(6) is_integer

Emp12hr(2) = 0
Emp12hr(4) = 0
Emp12hr(5) = 0
Emp12hr(6) = 0

TotalCost := sum(i in periods) (cost8hr*Emp8hr(i) + cost12hr*Emp12hr(i))

minimize(TotalCost)
```

An optimal schedule uses only 8-hour shift and costs $11200. The number of people starting at each shift is given below.

Period	8-hr Starts
12AM-4AM	2
4AM-8AM	8
8AM-12PM	8
12PM-4PM	5
4PM-8PM	6
8PM-12AM	6

2.9 Let $P(i, j)$ denote the amount of product i processed into product j, $Sold(i)$ indicate the amount of Product i sold, $Total(i)$ denote the amount of Product i available, and Raw denote the amount of raw material purchased. Our model is then

$$
\begin{aligned}
\max \quad & 10Sold(1) + 20Sold(2) + 30Sold(3) - ((25+1)Raw + P(1,2) + 2P(1,3) + 6P(2,3)) \\
\text{s.t.} \quad & \\
& 2Raw + 2P(1,2) + 3P(1,3) + P(2,3) \leq 25000 \qquad \text{(Labor)} \\
& Total(1) = 3Raw \\
& Total(1) = Sold(1) + P(1,2) + P(1,3) \\
& Total(2) = Raw + \tfrac{2}{3}P(1,2) \\
& Total(2) = Sold(2) + P(2,3) \\
& Sold(3) = \tfrac{1}{2}P(1,3) + \tfrac{3}{4}P(2,3) \\
& Sold(1) \leq 5000 \\
& Sold(2) \leq 4000 \\
& Sold(3) \leq 3000 \\
& \quad \text{All variables nonnegative.}
\end{aligned}
$$

Our optimal production schedule first purchases 3680 pounds of raw material. We sell 5000 ounces of product 1 and process 480 ounces into product 2 and 5560 ounces into product 3. We sell 4000 ounces of product 2 and 2780 ounces of product 3. This produces a profit of $117240.

2.12 We let Blends(i,j) denote the amount of crude oil i in gas j. Our model is given below.

```
  forall(i in nCrude) do
    Crude(i) = sum(j in nGas) Blends(i, j)
    Crude(i) <= maxCrude(i)
  end-do

  forall(j in nGas) do
    Gas(j) = sum(i in nCrude) Blends(i, j)
    Gas(j) >= minGas(j)
  end-do

! octane ratings

  forall(j in nGas) do
    sum(i in nCrude) OctaneRating(i) * Blends(i, j) >= minOctane(j) * Gas(j)
  end-do

! quality ratings

  forall(j in nGas) do
    sum(i in nCrude) QualityRating(i) * Blends(i, j) >= minQuality(j) * Gas(j)
  end-do

! objective
  Cost := sum(i in nCrude) PurchasePrice(i)*Crude(i) + 4 * sum(j in nGas) Gas(j)
```

```
   minimize(Cost)
```

The Optimal Cost is \$577,429. One solution uses 4428.57 barrels of Crude Oil 1, 0 barrels of Crude Oil 2, and 4571.43 barrels of Crude Oil 3, and it produces 4000 barrels of Gas 1 (2857.14 barrels of Crude Oil 1 and 1142.86 barrels of Crude Oil 3), 3000 barrels of Gas 2 (1285.71 barrels of Crude Oil 1 and 1714.29 barrels of Crude Oil 3), and 2000 barrels of Gas 3 (285.714 barrels of Crude Oil 1 and 1714.29 barrels of Crude Oil 3. The Octane ratings for Gasolines 1, 2, 3 are 87.5714, 90.1429, 92.7143, respectively, while the Quality ratings are 60, 70, and 80, respectively.

2.22 Let *employees*(m) denote the number of employees in month m, *hire*(m) and $fire(m)$ denote the number of employees hired and fired in month m, *produced*(m) be the number of sneakers produced in month m, $OT(m)$ the number of OT hours needed, and *inventory*(m) denote the amount kept in inventory at the end of month m. Our model is then

```
forall(m in months) do
   employees(m) = if(m > 1, employees(m-1), InitEmployees) + hire(m) - fire(m)
   inventory(m)= if(m > 1, inventory(m-1), InitInv) + produced(m) - demand(m)
   produced(m) <= ShoesperHour*(WorkHrs*employees(m) + OT(m))
   OT(m) <= OTmax*employees(m)

   hire(m) is_integer
   fire(m) is_integer
   produced(m) is_integer
end-do

Cost := sum(m in months) MonthlySalary * employees(m) +
        sum(m in months) HireCost * hire(m) +
        sum(m in months) FireCost * fire(m) +
        sum(m in months) OTPay * OT(m) +
        sum(m in months) StorageCost * inventory(m)

minimize(Cost)
```

The optimal cost is \$184,500. One solution is to never hire any additional employees and to fire 4 employees in month 1 and 1 in month 4. We produce 5000, 6400, 6600, 5000, 6000, and 5000 shoes in the respective months, which will produce inventory in months 2 (1400 shoes) and 4 (1000 shoes).

2.27 Let CCx and CCy denote the x,y coordinates of the container casting and CSx,CSy be the coordinates of the Container Storage. Our Mosel model is then

```
        PSx := 0
        PSy := 75
        FAx := 25
        FAy := 25
        SAx := 100
        SAy := 0
```

```
        CCCSxplus - CCCSxminus = CCx - CSx
        CCCSyplus - CCCSyminus = CCy - CSy

        CCPSxplus - CCPSxminus = CCx - PSx
        CCPSyplus - CCPSyminus = CCy - PSy

        CCFAxplus - CCFAxminus = CCx - FAx
        CCFAyplus - CCFAyminus = CCy - FAy

        CSFAxplus - CSFAxminus = CSx - FAx
        CSFAyplus - CSFAyminus = CSy - FAy

        CSSAxplus - CSSAxminus = CSx - FAx
        CSSAyplus - CSSAyminus = CSy - FAy

! different locations
        CCCSxplus + CCCSxminus + CCCSyplus + CCCSyminus >= 1
        CCPSxplus + CCPSxminus + CCPSyplus + CCPSyminus >= 1
        CCFAxplus + CCFAxminus + CCFAyplus + CCFAyminus >= 1
        CSFAxplus + CSFAxminus + CSFAyplus + CSFAyminus >= 1
        CSSAxplus + CSSAxminus + CSSAyplus + CSSAyminus >= 1

        Cost := 5.00 * (CCCSxplus + CCCSxminus + CCCSyplus + CCCSyminus) +
          2.15 * (CCPSxplus + CCPSxminus + CCPSyplus + CCPSyminus) +
          0.75 * (CCFAxplus + CCFAxminus + CCFAyplus + CCFAyminus) +
          0.85 * (CSFAxplus + CSFAxminus + CSFAyplus + CSFAyminus) +
          0.50 * (CSSAxplus + CSSAxminus + CSSAyplus + CSSAyminus)

        minimize(Cost)
```

The optimal location for the Container Casting Facility is (0, 74) and the optimal location of Container Storage Facility is (1, 74), which will cost \$211.20.

2.36 Let $PD(i, j)$ be the amount sent from plant i to distributor j and $DS(j, k)$ be the amount sent from distributor j to store k. Our network flow model is then

```
Cost := sum(i in plants, j in distributors)  PDCost(i,j) * PD(i,j) +
        sum(j in distributors, k in stores)  DSCost(j,k) * DS(j,k)

forall(i in plants) do
  PlantFlow(i) := sum(j in distributors) PD(i,j) <= Capacity(i)
end-do

forall(j in distributors) do
  DistributorFlow(j) := sum(i in plants) PD(i,j) = sum(k in stores) DS(j,k)
```

```
end-do

forall(k in stores) do
  CustomerFlow(k) :=          sum(j in distributors) DS(j,k) >= Demand(k)
end-do

minimize(Cost)
```

An optimal solution sends 1000 bottles of beer from plant 1 to distributor 1, 750 bottles from plant 2 to distributor 2, and 350 bottles from plant 3 to distributor 2. From distributor 1 we send 600 bottles to store 2 and 400 bottles to store 3, while we send 700 bottles from distributor 2 to store 1 and 400 bottles from distributor 2 to store 3. The optimal cost is $37,100.

Chapter 3

3.1 Our model is as follows, with flow variables $PW(i, j)$ and $WC(j, k)$ and binary variables $OpenPlant(i)$, $OpenWarehouse(j)$ designating whether a given plant or warehouse is to be opened.

```
forall(i in Plants) do
  PlantFlow(i) := sum(j in Warehouses) PW(i,j) <= Capacity(i)*OpenPlant(i)
  OpenPlant(i) is_binary
end-do

forall(j in Warehouses) do
  WarehouseFlow(j) := sum(i in Plants) PW(i,j) = sum(k in Customers) WC(j,k)
  WHOpen(j) := sum(i in Plants) PW(i,j) <=
                      (sum(i in Plants) Capacity(i)) * OpenWarehouse(j)
  OpenWarehouse(j) is_binary
end-do

forall(k in Customers) do
  CustomerFlow(k) :=        sum(j in Warehouses) WC(j,k) >= Demand(k)
end-do

ShipCost := sum(i in Plants, j in Warehouses) PWCost(i,j) * PW(i,j) +
            sum(j in Warehouses, k in Customers) WCCost(j,k) * WC(j,k)
SetupCost:= sum(i in Plants) SetupPlant(i) * OpenPlant(i) +
            sum(j in Warehouses) SetupWarehouse(j) * OpenWarehouse(j)

TotCost := ShipCost + SetupCost

minimize(TotCost)
```

An optimal solution costs $700,500 and opens Plants 2, 3, and 5 along with Warehouses 2 and 3. We ship 200 units from Plant 2 to Warehouse 2, 300 units from Plant 3 to Warehouse 3, and 400 units from Plant 5 to Warehouse 5. We then ship 200 units from Warehouse 2 to Customer 1, 300 units from Warehouse 3 to Customer 2, 150 units from Warehouse 3 to Customer 3, and 250 units from Warehouse 3 to Customer 4.

3.3 Let $Workers(i, j)$ denote how many workers are assigned to Product i on Line j, and $OpenLine(j)$ designate whether Line j is to be opened. Our model is then as follows.

```
TotWorkers := sum(i in Products, j in Lines) Workers(i,j) <= maxWorkers

forall(j in Lines) do
  WL(j) := sum(i in Products) Workers(i,j) <= maxWorkers * OpenLine(j)
  OpenLine(j) is_binary
end-do

forall(i in Products) do
  Prod(i) := sum(j in Lines) ProdAmts(i,j) * Workers(i,j) >= Demand(i)
  forall(j in Lines) Workers(i,j) is_integer
end-do

TotCost := sum(i in Products, j in Lines) WorkerCost(j) * Workers(i,j) +
           sum(j in Lines) SetupCost(j) * OpenLine(j)

minimize(TotCost)
```

The optimal cost is \$30,500, obtained by opening Line 2, with 5 workers on Product 2 and 8 workers on Product 3, and Line 3, with 5 workers on Product 1 and 2 workers on Product 2. This solution produces 600 units of Product 1, 810 units of Product 2, and 1000 units of Product 3.

3.7 Let $x(i, j)$ denote whether reactor i is set to setting j. Our model is then given below.

```
forall(i in Reactors) do
  SelectSetting(i) := sum(j in Settings) x(i,j) <= 1
  forall(j in Settings) do
    x(i,j) is_binary
  end-do
end-do

AnnualProduction := sum(i in Reactors, j in Settings) Pounds(i,j) * x(i,j)
                    >= Demand

TotCost := sum(i in Reactors, j in Settings) Cost(i,j) * x(i,j)

minimize(TotCost)
```

The optimal cost is \$425, found by setting Reactor 1 to setting 4, Reactor 2 to setting 1, Reactor 3 to setting 2, and Reactor 4 to setting 4.

3.10 The optimal tour is $1-3-2-7-6-4-8-5-1$ with length 67. The MTZ model is given below.

```
! Objective: minimize the total delay
 TotalDist:= sum(i,j in Holes | i<>j) Dist(i,j)*x(i,j)

! Visit every city once
```

```
 forall(i in Holes) sum(j in Holes | i<>j) x(i,j) = 1
 forall(j in Holes) sum(i in Holes | i<>j) x(i,j) = 1

 forall(i,j in Holes | i<>j) x(i,j) is_binary

 forall(i, j in Holes | i <> 1 and j <> 1 and i <> j)
         uij(i,j) := u(j) >= u(i) + 1 - (NHoles-1)*(1 - x(i,j))

 u(1) = 1
 forall(i in Holes | i <> 1) do
         u(i) >= 2
         u(i) <= NHoles
 end-do

! Solve the problem
 minimize(TotalDist)
```

3.22 Let *Build*(i) denote whether Depot i is built and $x(i,j)$ denote whether customer j's demand is satisfied by depot i. Our model is then as follows:

```
forall(j in Customers)
  sum(i in Depots) x(i, j) = 1

forall(i in Depots) do
  sum(j in Customers) Demands(j) * x(i,j) <= Capacity(i)

  Build(i) is_binary
end-do

forall(i in Depots, j in Customers) do
  x(i, j) <= Build(i)

  x(i,j) is_binary
end-do

TotCost := sum(i in Depots) Cost(i) * Build(i) +
           sum(i in Depots, j in Customers) DelivCosts(i,j) * Demands(j) * x(i,j)

minimize(TotCost)
```

Our optimal cost is $95,022,025.00. We are to build Depots 5 and 6, with Depot 5 servicing customers 5, 6, 9, 10, and 11, while Depot 6 servicing customers 1, 2, 3, 4, 7, 8.

3.36 We will assume that the coefficients a_j and b are integers, and that M is chosen so that

$$-M \leq \sum_{j=1}^{n} a_j - b \leq M.$$

The inequalities

$$\sum_{j=1}^{n} a_j \leq b + M(1-y)$$
$$b + 1 \leq \sum_{j=1}^{n} a_j + My$$

satisfy the condition. When $y = 1$ we must have $\sum_{j=1}^{n} a_j \leq b$, while when $y = 0$ we must have $\sum_{j=1}^{n} a_j \geq b + 1$.

Chapter 4

4.1 **Errata: Distances $d_{25} = d_{52}$ should be 35 instead of 40.** Since our total capacity is 335, we need 3 trucks to satisfy these routes. The initial tours included routes $1-2-3-1$, $1-4-5-1$, $1-6-7-1$; unfortunately, the demand on the first route exceeds that truck capacity. Our subtour elimination constraint is then $x_{23} \leq 2 - 2 = 0$, since at least two different trucks are needed for customers $2, 3$. Adding this constraint generates the tour $1-2-4-1$, which again violates the truck capacity. Adding the constraint $x_{24} \leq 0$ and resolving generates the feasible tours $1-2-7-1$, $1-4-6-1$, $1-3-5-1$ which has length 175 miles.

4.4 The is the Set Covering Location Model (Integer Program 4.2). We need 2 facilities: facility 5, which can service customers $\{1, 2, 4, 5, 8, 10\}$ and facility 6, which can service customers $\{1, 2, 3, 6, 7, 8, 9, 10\}$.

4.7 This is a p-median problem (Integer Program 4.5). If we locate the facility at 6, the total weighted distance is 2610.

4.9 This is a Fixed-Charge Location Problem (Integer Program 4.6). The optimal cost is \$9335 incurred by opening facilities 6 and 7, where 6 serves customers $\{1, 2, 5, 6, 8, 9, 10\}$ and 7 serves $\{3, 4, 7\}$.

Chapter 5

Note: None of the algorithms presented here are the only solutions. Acceptable heuristics need only produce solutions based upon "reasonable" approaches to the problem.

5.1 Ordering the variables according to their $\frac{c_k}{a_k}$ ratios gives the order $x_4, x_3, x_7, x_5, x_2, x_8, x_6, x_1$. In the first 4 iterations, we set $x_4 = 1$, $x_3 = 1$, $x_7 = 1$, $x_5 = 1$. After fixing $x_5 = 1$ we find that there is only $\widehat{b} = 4$ units available in the right-hand side, which means that x_1 and x_8 must be set to 0. In the next iteration we fix $x_2 = 1$, which then fixes $x_6 = 0$. Our greedy heuristic generates the solution $(0, 1, 1, 1, 1, 0, 1, 0)$.

5.2 One approach starts with the solution $\mathbf{x} = (1, 1, \dots, 1)$, where the variables are ordered so that

$$\frac{c_1}{a_1} \geq \frac{c_2}{a_2} \geq \cdots \geq \frac{c_n}{a_n}.$$

At iteration k we set $x_k = 0$ if we can maintain feasibility. In our sample problem, the variables are considered in the order $x_1, x_6, x_2, x_8, x_5, x_7, x_3, x_4$, producing the solution $(0, 0, 1, 1, 1, 0, 1, 0)$.

5.4 Our initial vector $\mathbf{d} = (5, 2, 3, 4, 2, 2, 3, 4, 4, 3, 2, 2)$. We set $x_1 = 1$ first, and so our updated $\mathbf{d}$ is

$$\mathbf{d} = (-, 1, 0, 2, 0, 2, 0, 1, 1, 3, 2, 1).$$

Setting $x_{10} = 1$ next we get

$$\mathbf{d} = (-, 1, 0, 0, 0, 0, 0, 1, 1, -, 0, 0).$$

If we set $x_2 = 1$ next, we get $\mathbf{d} = \mathbf{0}$, and so our greedy solution is $(1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0)$.

5.9

(a) The Nearest Neighbor Heuristic finds the ordering

$$3 \to 2 \to 4 \to 6 \to 7 \to 8 \to 5 \to 1 \to 3$$

with distance 68.

(b) The Cheapest Insertion Heuristic finds the ordering

$$1 \to 3 \to 2 \to 4 \to 7 \to 6 \to 8 \to 5 \to 1$$

with distance 69.

(c) For the Nearest Neighbor solution, exchanging the edges $(2,4)$ and $(7,8)$ with $(2,7)$ and $(4,8)$ results in a tour of length 67. No other improvements are possible by exchange. For the Cheapest Insertion solution, exchanging the edges $(4,7)$ and $(6,8)$ with $(4,6)$ and $(7,8)$ results in the Nearest Neighbor solution, which then can be improved one more time. Each results in the ordering

$$3 \to 2 \to 7 \to 6 \to 5 \to 8 \to 5 \to 1 \to 3$$

with distance 67.

(d) The optimal solution is the 2-opt solution in part (c). It needed 0 subtour elimination constraints:

5.11 One approach is to mimic the Cheapest Insertion Heuristic. For each pair of initial hole locations i_1, i_2, the next hole drilled is the cheapest inserted into either of the current subtours generated.

5.16 One approach is to schedule, at time t, the job that minimizes $\max\{t + p_j, d_j\}$ among all unscheduled jobs j. For example, suppose $\widehat{J}$ is the set of unscheduled jobs (so that initially $\widehat{J} = \{1, \ldots, 9\}$). Since

$$\min_{j \in \widehat{J}} \max\{0 + p_j, d_j\} = 23$$

for job $j = 6$, job 6 is schedule first and $\widehat{J} = \widehat{J} - \{6\}$. Now starting at time $t = 8$ (since job 6 has $p_6 = 8$),

$$\min_{j \in \widehat{J}} \max\{8 + p_j, d_j\} = 35$$

for job $j = 1$. Continuing this yields the job sequence $6 - 1 - 3 - 8 - 2 - 5 - 7 - 9 - 4$.

Chapter 6

6.1(i) Note that $\nabla f = (3, 4, -6)$ for every $\mathbf{x}$ and we are maximizing our objective.

(a) Not an improving direction since $\nabla f^T \mathbf{d} = -7 < 0$.

(b) Is an improving direction since $\nabla f^T \mathbf{d} = 15 > 0$.

(c) Is an improving direction since $\nabla f^T \mathbf{d} = 2 > 0$.

6.2 (a) Starting at $\mathbf{x}^0 = (1, 0, -2)$,

$$\begin{aligned}
\mathbf{x}^1 &= \mathbf{x}^0 + \lambda_1 \mathbf{d}^1 \\
&= (1, 0, -2) + 2(3, 2, 1) \\
&= (7, 4, 0). \\
\mathbf{x}^2 &= \mathbf{x}^1 + \lambda_2 \mathbf{d}^2 \\
&= (7, 4, 0) + 4(1, 0, -3) \\
&= (11, 4, -12). \\
\mathbf{x}^3 &= \mathbf{x}^2 + \lambda_3 \mathbf{d}^3 \\
&= (11, 4, -12) + \frac{1}{2}(-1, 4, 0) \\
&= (\frac{21}{2}, 6, -12).
\end{aligned}$$

6.3 (a) We have $\nabla f = (6x - 2y, 2y - 2x)$, and so $\nabla f(1, 3) = (0, 4)$ with search direction $\mathbf{d} = -\nabla f$. Our next solution is then

$$\mathbf{x}^1 = (1, 3) + \lambda(0, -4) = (1, 3 - 4\lambda).$$

To find our step size, we find the smallest positive local minimum of

$$\begin{aligned}
g(\lambda) &= 3(1)^2 - 2(1)(3 - 4\lambda) + (3 - 4\lambda)^2 - 10 \\
&= 16\lambda^2 - 16\lambda - 4.
\end{aligned}$$

Since $g'(\lambda) = 32\lambda - 16 = 0$ at $\lambda = \frac{1}{2}$, this is our step size.

6.5 (b) Our next solution is of the form $\mathbf{x}' = (3 + 2\lambda, 1 - \lambda, 5 - 3\lambda)$, which remains $\geq \mathbf{0}$ as long as $\lambda \leq 1$ and $\lambda \leq \frac{5}{3}$. Putting this solution into the constraint yields

$$\begin{aligned}
2(3 + 2\lambda) + 4(1 - \lambda) - (5 - 3\lambda) &= 5 + 3\lambda \\
&\leq 20,
\end{aligned}$$

which implies that $\lambda \leq 5$ in order to remain feasible. Thus, the maximum step size in this direction is

$$\widehat{\lambda} = \min\{1, \tfrac{5}{3}, 5\} = 1.$$

6.10 (a) Since only the first constraint and the bound $x_2 \geq 0$ is active we need

$$\begin{array}{rcrcl} 2d_1 & & & + d_3 & = 12 \\ & & d_2 & & \leq 0 \end{array}$$

6.1. One direction is given in Exercise 6.21. To show that a function that is both convex and concave must be affine means that we must show that the function

$$h(\mathbf{x}) = f(\mathbf{x}) - f(\mathbf{0})$$

is linear, i.e. that it satisfies

$$h(\alpha\mathbf{x} + \beta\mathbf{y}) = \alpha h(\mathbf{x}) + \beta h(\mathbf{y}).$$

Note that $f(\mathbf{x})$ satisfies

$$f(\lambda\mathbf{x} + (1-\lambda)\mathbf{y}) = \lambda f(\mathbf{x}) + (1 - \lambda f(\mathbf{y})$$

for all $0 \leq \lambda \leq 1$. This can be done by showing each of the following:

(a) $h(\frac{1}{2}\mathbf{x} + \frac{1}{2}\mathbf{y}) = \frac{1}{2}h(\mathbf{x}) + \frac{1}{2}h(\mathbf{y})$.

(b) $h(\alpha\mathbf{x}) = \alpha h(\mathbf{x})$ for $0 \leq \alpha \leq 1$.

(c) $h(\mathbf{x} + \mathbf{y}) = h(\mathbf{x}) + h(\mathbf{y})$.

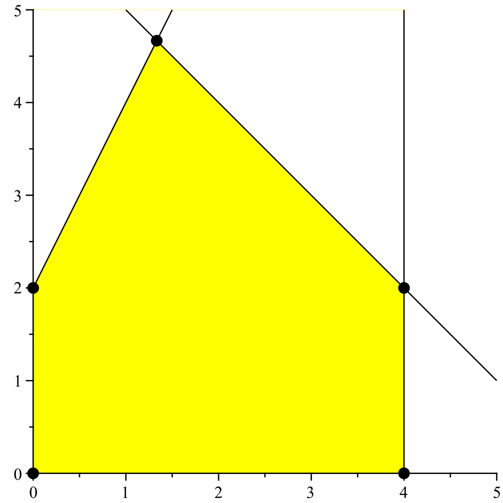
(d) $h(-\mathbf{x}) = -h(\mathbf{x})$.

(e) $h(\beta\mathbf{x}) = \beta h(\mathbf{x})$ for $\beta > 1$.

It follows from these notions that $h(\mathbf{x})$ is linear.

Chapter 7

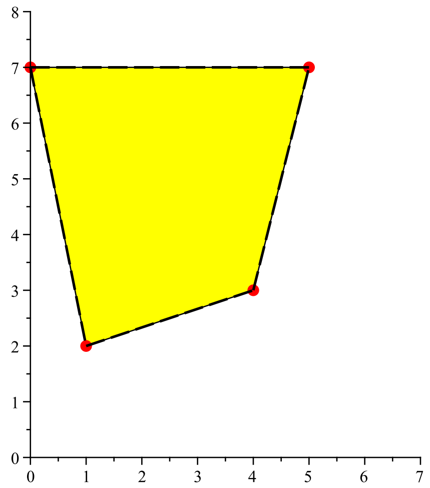
7.1 The extreme points are $(0,0)$, $(0,2)$, $(4,0)$, $(4,2)$, and $(\frac{4}{3}, \frac{14}{3})$, as noted below.



7.5 The convex hull of P is described by the constraints

$$\begin{aligned} 4x - \ \ y &\le 13 \\ x - 3y &\le -5 \\ 5x + \ \ y &\ge 7 \\ y &\le 7. \end{aligned}$$

The extreme points are exactly the elements of P



7.7 (a) One possible objective function (note there are many) for each extreme point is given below:

Extreme Point	Objective
$(0,7)$	$\max y - x$
$(1,2)$	$\min x + y$
$(4,3)$	$\max x - y$
$(5,7)$	$\max x + y$

7.14 (c)

$$
\begin{aligned}
\max \quad & 4x_1 + 2x_2 + 7x_3 \\
\text{s.t.} \quad & \\
& 2x_1 - x_2 + 4x_3 + s_1 \phantom{{}-s_2} = 18 \\
& 4x_1 + 2x_2 + 5x_3 \phantom{{}+s_1} - s_2 = 10 \\
& x_1, x_2, x_3, s_1, s_2 \geq 0.
\end{aligned}
$$

7.15 (c)

$$\max \begin{bmatrix} 4 & 2 & 7 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \end{bmatrix}$$

s.t.

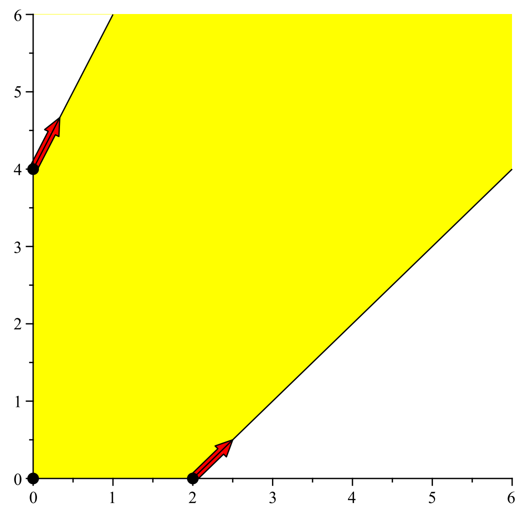
$$\begin{bmatrix} 2 & -1 & 4 & 1 & 0 \\ 4 & 2 & 5 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 18 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

7.21 (c) Basic feasible solutions are:

Basic Feasible Solution	Basic Variables	Nonbasic variables
$(0, 0, \frac{9}{2}, 0, \frac{25}{2})$	x_3, s_2	x_1, x_2, s_1
$(0, 0, 2, 10, 0)$	x_3, s_1	x_1, x_2, s_2
$(0, 5, 0, 23, 0)$	x_2, s_1	x_1, x_3, s_1
$(9, 0, 0, 0, 26)$	x_1, s_2	x_2, x_3, s_2
$(\frac{5}{2}, 0, 0, 13, 0)$	x_1, s_1	x_2, x_3, s_2.

7.33 The feasible region is given below.



(a) The extreme points are $(0, 0)$, $(0, 4)$, and $(2, 0)$.

(b) The extreme directions are $(\frac{1}{3}, \frac{2}{3})$ and $(\frac{1}{2}, \frac{1}{2})$.

(c) The objective vector $\mathbf{c} = (c_x, c_y)$ for which there is a finite optimal solution must satisfy $\mathbf{c}^T\mathbf{d} \leq 0$ for every unbounded direction, or

$$\frac{1}{2}c_x + \frac{1}{2}c_y \leq 0$$
$$\frac{1}{3}c_x + \frac{2}{3}c_y \leq 0$$

(d) The objective vector $\mathbf{c}$ must satisfy $\mathbf{c}^T\mathbf{d} > 0$ for some unbounded direction $\mathbf{d}$, which implies either $\frac{1}{2}c_x + \frac{1}{2}c_y > 0$ or $\frac{1}{3}c_x + \frac{2}{3}c_y > 0$.

Chapter 8

8.1 After adding slack variables s_1, s_2, s_3 our initial basic feasible solution is $\mathbf{x}^0 = (0,0,0,2,3,8)$ with $\mathcal{B} = \{s_1, s_2, s_3\}$.

Iteration 1: The simplex direction for variable x_1 is

$$\mathbf{d}^{x_1} = (1,0,0,d_{s_1},d_{s_2},d_{s_3}) = (1,0,0,-1,-2,-6),$$

where $\mathbf{d}_B = (d_{s_1},d_{s_2},d_{s_3})$ is found by solving

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}d_{s_1}\\d_{s_2}\\d_{s_3}\end{bmatrix} = -\begin{bmatrix}1\\2\\6\end{bmatrix}.$$

Its reduced cost is $\bar{c}_{x_1} = 8$, and hence it is an improving direction. The simplex direction for variable x_2 is

$$\mathbf{d}^{x_2} = (0,1,0,d_{s_1},d_{s_2},d_{s_3}) = (0,1,0,-1,-3,-6),$$

where $\mathbf{d}_B = (d_{s_1},d_{s_2},d_{s_3})$ is found by solving

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}d_{s_1}\\d_{s_2}\\d_{s_3}\end{bmatrix} = -\begin{bmatrix}1\\3\\6\end{bmatrix}.$$

Its reduced cost is $\bar{c}_{x_1} = 9$, and hence it is an improving direction. The simplex direction for variable x_3 is

$$\mathbf{d}^{x_3} = (0,0,1,d_{s_1},d_{s_2},d_{s_3}) = (0,0,1,-2,-4,-2),$$

where $\mathbf{d}_B = (d_{s_1},d_{s_2},d_{s_3})$ is found by solving

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}d_{s_1}\\d_{s_2}\\d_{s_3}\end{bmatrix} = -\begin{bmatrix}2\\4\\2\end{bmatrix}.$$

Its reduced cost is $\bar{c}_{x_1} = 3$, and hence it is an improving direction. Using Dantzig's rule, we choose x_2 as our entering variable. By the Ratio Test, our step size is

$$\begin{aligned}\lambda_{\max} &= \min\left\{\frac{2}{-(-1)}, \frac{3}{-(-3)}, \frac{8}{-(-6)}\right\}\\ &= 1.\end{aligned}$$

Our leaving variable is then s_2. Our new basic feasible solution is

$$\mathbf{x}^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \\ 3 \\ 8 \end{bmatrix} + 1 \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \\ -3 \\ -6 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 2 \end{bmatrix}$$

with basis $\mathcal{B} = \{x_2, s_1, s_3\}$.

Iteration 2: The simplex direction for variable x_1 is

$$\mathbf{d}^{x_1} = (1, d_{x_2}, 0, d_{s_1}, 0, d_{s_3}) = (1, -\tfrac{2}{3}, 0, -\tfrac{1}{3}, 0, -2),$$

where $\mathbf{d}_B = (d_{x_2}, d_{s_1}, d_{s_3})$ is found by solving

$$\begin{bmatrix} 1 & 1 & 0 \\ 3 & 0 & 0 \\ 6 & 0 & 1 \end{bmatrix} \begin{bmatrix} d_{x_2} \\ d_{s_1} \\ d_{s_3} \end{bmatrix} = - \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}.$$

Its reduced cost is $\bar{c}_{x_1} = 8 - \frac{2}{3}(9) = 2$, and hence it is an improving direction. The simplex direction for variable x_3 is

$$\mathbf{d}^{x_3} = (0, d_{x_2}, 1, d_{s_1}, 0, d_{s_3}) = (0, -\tfrac{4}{3}, 1, -\tfrac{2}{3}, 0, 6),$$

where $\mathbf{d}_B = (d_{x_2}, d_{s_1}, d_{s_3})$ is found by solving

$$\begin{bmatrix} 1 & 1 & 0 \\ 3 & 0 & 0 \\ 6 & 0 & 1 \end{bmatrix} \begin{bmatrix} d_y \\ d_{s_1} \\ d_{s_3} \end{bmatrix} = - \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}.$$

Its reduced cost is $\bar{c}_{x_3} = 5 + (9)(-\frac{4}{3}) = -7$, and hence it is not an improving direction. The simplex direction for variable s_2 is

$$\mathbf{d}^{s_2} = (0, d_{x_2}, 0, d_{s_1}, 1, d_{s_3}) = (0, -\tfrac{1}{3}, 0, \tfrac{1}{3}, 1, 2),$$

where $\mathbf{d}_B = (d_{x_2}, d_{s_1}, d_{s_3})$ is found by solving

$$\begin{bmatrix} 1 & 1 & 0 \\ 3 & 0 & 0 \\ 6 & 0 & 1 \end{bmatrix} \begin{bmatrix} d_y \\ d_{s_1} \\ d_{s_3} \end{bmatrix} = - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

Its reduced cost is $\bar{c}_{x_3} = 0 + (9)(-\frac{1}{3}) = -3$, and hence it is not an improving direction. Using Dantzig's rule, we choose x_1 as our entering variable. By the Ratio Test, our step size is

$$\begin{aligned} \lambda_{\max} &= \min \left\{ \frac{1}{-(-\frac{2}{3})}, \frac{1}{-(-\frac{1}{3})}, \frac{2}{-(-2)} \right\} \\ &= 1. \end{aligned}$$

Our leaving variable is then s_3. Our new basic feasible solution is

$$\mathbf{x}^2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 2 \end{bmatrix} + 1 \begin{bmatrix} 1 \\ -\frac{2}{3} \\ 0 \\ -\frac{1}{3} \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{3} \\ 0 \\ \frac{2}{3} \\ 0 \\ 0 \end{bmatrix}$$

with basis $\mathcal{B} = \{x_1, x_2, s_1\}$.

Iteration 3: The simplex direction for variable x_3 is

$$\mathbf{d}^{x_3} = (d_{x_1}, d_{x_2}, 1, d_{s_1}, 0, 0) = (3, -\tfrac{10}{3}, 1, -\tfrac{5}{3}, 0, 0),$$

where $\mathbf{d}_B = (d_{x_1}, d_{x_2}, d_{s_1})$ is found by solving

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 0 \\ 6 & 6 & 0 \end{bmatrix} \begin{bmatrix} d_{x_1} \\ d_{x_2} \\ d_{s_1} \end{bmatrix} = - \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}.$$

Its reduced cost is $\bar{c}_{x_3} = 5 + 8(3) + 9(-\frac{10}{3}) = -1$, and hence it is not an improving direction. The simplex direction for variable s_2 is

$$\mathbf{d}^{s_2} = (d_{x_1}, d_{x_2}, 0, d_{s_1}, 1, 0) = (1, -1, 0, 0, 1, 0),$$

where $\mathbf{d}_B = (d_{x_1}, d_{x_2}, d_{s_1})$ is found by solving

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 0 \\ 6 & 6 & 0 \end{bmatrix} \begin{bmatrix} d_{x_1} \\ d_{x_2} \\ d_{s_1} \end{bmatrix} = - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

Its reduced cost $\bar{c}_{s_2} = 0 + 8(1) + 9(-1) = -1$, and hence it is not an improving direction. The simplex direction for variable s_3 is

$$\mathbf{d}^{s_3} = (d_{x_1}, d_{x_2}, 0, d_{s_1}, 0, 1) = (-\tfrac{1}{2}, \tfrac{1}{3}, 0, \tfrac{1}{6}, 0, 1),$$

where $\mathbf{d}_B = (d_{x_1}, d_{x_2}, d_{s_1})$ is found by solving

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 0 \\ 6 & 6 & 0 \end{bmatrix} \begin{bmatrix} d_{x_1} \\ d_{x_2} \\ d_{s_1} \end{bmatrix} = - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Its reduced cost is $\bar{c}_{s_3} = 0 + 8(-\frac{1}{2}) + 9(\frac{1}{3}) = -1$, and hence it is not an improving direction. Since there are no improving simplex directions, our current solution $\mathbf{x}^2 = (1, \frac{1}{3}, 0, \frac{2}{3}, 0, 0)$ is optimal.

8.1.

(a) The Phase I linear program is

$$\begin{array}{ll} \min & a_2 + a_3 \\ \text{s.t.} & \\ & x_1 + x_2 + s_1 = 18 \\ & 2x_1 - x_3 + a_2 = 12 \\ & 3x_2 + 5x_3 - s_3 + a_3 = 15 \\ & x_1, x_2, x_3, s_1, s_3, a_2, a_3 \geq 0 \end{array}$$

(b) Starting with the basic variables $\mathcal{B} = \{s_1, a_2, a_3\}$, we have the following simplex method iterations when solving the Phase I problem:

Iteration	Basis $\mathcal{B}$	$\overline{\mathbf{c}}_N$	Entering var	$\mathbf{d}_B^e$	Leaving var
1	$\{s_1, a_2, a_3\}$	$(-2, -3, -4, 1)$	x_3	$(0, 1, -5)$	a_3
2	$\{x_3, s_1, a_2\}$	$(-2, -\frac{3}{5}, \frac{1}{5}, \frac{4}{5})$	x_1	$(-1, -2, 0)$	a_2
3	$\{x_1, x_3, s_1\}$	$(0, 0, 1, 1)$			

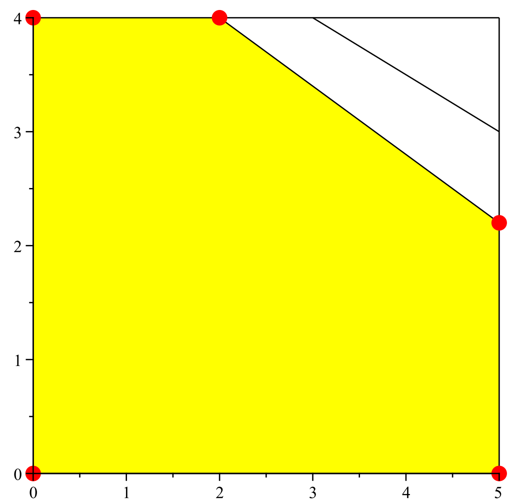
The optimal solution the the Phase I problem has $\mathbf{x}_B = (x_1, x_3, s_1) = (\frac{15}{2}, 3, \frac{21}{2})$ with value 0. Our initial problem has a feasible solution.

(c) Starting from the Phase I solution (after removing a_2, a_3 from the problem), we have the following simplex method iterations:

Iteration	Basis $\mathcal{B}$	$\overline{\mathbf{c}}_N$	Entering var	$\mathbf{d}_B^e$	Leaving var
1	$\{x_1, x_3, s_1\}$	$(-5, 2)$	s_3	$(\frac{1}{10}, \frac{1}{5}, -\frac{1}{10})$	s_1
2	$\{x_1, x_3, s_3\}$	$(-19, -20)$			

The optimal solution the the Phase II problem (and to the original problem) has $\mathbf{x}_B = (x_1, x_3, s_3) = (18, 24, 105)$.

8.18 The feasible region is given below.



The extended basic feasible solutions and partitions are as follows:

Extended BFS	$(\mathcal{B}, \mathcal{L}, \mathcal{U})$
$(0, 0, 11, 26)$	$(\{s_1, s_2\}, \{x, y\}, \emptyset)$
$(5, 0, 6, 11)$	$(\{s_1, s_2\}, \{y\}, \{x\})$
$(5, 2.2, 1.6, 0)$	$(\{y, s_1\}, \{s_2\}, \{x\})$
$(2, 4, 1, 0)$	$(\{x, s_1\}, \{s_2\}, \{y\})$
$(0, 4, 3, 6)$	$(\{s_1, s_2\}, \{x\}, \{y\})$

8.20 The bounded simplex method iterations are given below. We start with the extended basic feasible solution $(0, 0, 11, 26)$ (after adding slack variables).

Iteration	$(\mathcal{B}, \mathcal{L}, \mathcal{U})$	$\mathbf{x}_B$	$\overline{\mathbf{c}}_N$	Entering var	$\mathbf{d}_B^e$	Leaving var
1	$(\{s_1, s_2\}, \{x, y\}, \emptyset)$	$(11, 26)$	$(2, 3)$	y	$(-2, 5)$	y
2	$(\{s_1, s_2\}, \{x\}, \{y\})$	$(3, 6)$	$(2, 3)$	x	$(-1, -3)$	s_2
3	$(\{x, s_1\}, \{s_2\}, \{y\})$	$(2, 1)$	$(-\frac{1}{3}, -\frac{2}{3})$	y	$(\frac{5}{3}, \frac{1}{3})$	x
4	$(\{y, s_1\}, \{s_2\}, \{x\})$	$(\frac{11}{5}, \frac{8}{5})$	$(\frac{1}{5}, -\frac{3}{5})$			

The optimal solution is $(5, \frac{11}{5}, \frac{8}{5}, 0)$.

Chapter 9

9.1

$$
\begin{aligned}
\min \quad & 20y_1 + 8y_2 + 10y_3 \\
\text{s.t.} \quad & \\
& 2y_1 + 3y_3 \geq 8 \\
& 3y_1 + y_2 + y_3 \geq 20 \\
& 2y_1 + y_2 \geq 4 \\
& y_1, y_2 \geq 0, y_3 \leq 0
\end{aligned}
$$

9.6

(a) At this solution the basic variables are $\mathcal{B} = \{x_1, x_2, x_3\}$ the reduced cost vector is then

$$
\begin{aligned}
\bar{\mathbf{c}}_N^T &= \mathbf{c}_N^T - \mathbf{c}_B^T B^{-1} N \\
&= \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 9 & 14 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 \\ 5 & 4 & 1 \\ 0 & 2 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} -2 & -1 & -4 \end{bmatrix}
\end{aligned}
$$

(b) The dual is

$$
\begin{aligned}
\min \quad & 6y_1 + 12y_2 + 5y_3 \\
\text{s.t.} \quad & \\
& 2y_1 + 5y_2 \geq 9 \\
& y_1 + 4y_2 + 2y_3 \geq 14 \\
& 3y_1 + y_2 \geq 7 \\
& y_1, y_2, y_3 \geq 0
\end{aligned}
$$

(c) The optimal dual solution is

$$
\mathbf{c}_B^T B^{-1} = \begin{bmatrix} 2 & 1 & 4 \end{bmatrix}
$$

Chapter 10

10.1 (a) The optimal solution occurs at the intersection of two constraints whose slopes are $-\frac{3}{2}$ and $-\frac{1}{2}$. Based upon this, the range of values are $5 \le c_x \le 15$ and $6 \le c_y \le 18$.

(b) First consider constraint 1, which we alter to $x + 2y \le 30 + \Delta$. As we increase Δ from 0, the optimal solution remains at the intersection of the first two constraints until the bound $x \ge 0$ becomes active. Thus the largest value of Δ for which the optimal solution lies at the intersection of the first two constraints is found by solving

$$\begin{aligned} x + 2y &= 30 + \Delta \\ 3x + 2y &= 42 \\ x \qquad &= 0, \end{aligned}$$

which gives $\Delta = 12$. Similarly, decreasing the value of Δ from 0 has the optimal solution at the intersection of the first two constraints until the third constraint is also active. Solving these equations yields $\Delta = -8$. Thus, for the optimal solution to be at the intersection of the first two constraints, we must have that the right-hand side value of the first constraint be between $30 - 8 = 22$ and $30 + 12 = 42$. For all values of Δ in this range, the optimal solution is at $x = 6 - \frac{1}{2}\Delta$ and $y = 12 + \frac{3}{4}\Delta$, and the objective value is $174 + 3\Delta$. This indicates that the shadow price for this constraint is 3.

Similar analysis for the second constraint yields the range $42 - 12 = 30 \le b_2 \le 42 + \frac{72}{7}$ with a shadow price of 2. The third constraint is not active at the optimal solution, so its shadow price is 0. This shadow price is correct for all $b_3 \ge 24 - 18 = 6$.

(c) Using the basis $\mathcal{B} = \{x, y, s_3\}$ we find the following information:

$$\mathbf{y} = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}, \quad \bar{\mathbf{c}}_N = \begin{bmatrix} -3 \\ -2 \end{bmatrix}, \quad B^{-1} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & 0 \\ \frac{3}{4} & -\frac{1}{4} & 0 \\ \frac{9}{4} & -\frac{7}{4} & 1 \end{bmatrix}, \quad B^{-1}N = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{3}{4} & -\frac{1}{4} \\ \frac{9}{4} & -\frac{7}{4} \end{bmatrix}.$$

The shadow prices are exactly the values in $\mathbf{y}$. The range objective coefficient changes Δc_k use the k^{th} row of $B^{-1}N$ for the basic variables. Thus we have

$$\begin{aligned} \frac{-2}{1/2} &= -4 \le \Delta c_x \le \frac{-3}{-1/2} = 6 \\ \frac{-3}{3/4} &= 4 \le \Delta c_y \le \frac{-2}{-1/4} = 8. \end{aligned}$$

The right-hand side ranges use the columns of b^{-1} in their calculations. We then have

$$\max\{\frac{-12}{3/4}, \frac{-18}{9/4}\} = -8 \le \Delta b_1 \le \frac{-6}{-1/2} = 12$$
$$\frac{-6}{1/2} = -12 \le \Delta b_2 \le \min\{\frac{-12}{-1/4}, \frac{-18}{-7/4}\} = \frac{72}{7}$$
$$\frac{-18}{1} = -18 \le \Delta b_3 < \infty.$$

(d) Since $3x - y \le 24$ is not active at the optimal solution, its removal does not change the optimal solution. If we remove $x + 2y \le 30$, the optimal solution $(0, 21)$.

10.3 (a) Produce 80 Q75s and 75 Q100s.

(b) We can individually change the price of Q75s up to \$93.75 and the price of Q100s up to \$ 82.50 without altering the optimal production amounts.

(c) We can add up to 11.67 hours. Given the shadow price of 22.143, we would want to pay no more than this amount for these extra hours.

(d) Profit changes by 2.143 (number of hours), since both amounts are within our valid ranges.

(e) Adding such a constraint $Q100 \ge Q75 + 10$ requires resolving the problem to see its affect.

10.9 Solving this problem for $\theta = 0$ we find the solution $(6, 12)$ with $\mathcal{B} = \{x, y, s_3\}$. If we include $\theta(2x + y)$ in the objective, this basic feasible solution has reduced cost

$$\overline{\mathbf{c}}_N = \begin{bmatrix} -3 + \frac{1}{4}\theta & -2 - \frac{3}{4}\theta \end{bmatrix}.$$

Thus our current solution remains optimal for $-\frac{8}{3} \le \theta \le 12$. When $\theta = 12$ we have the new optimal basis $\mathcal{B} = \{x, y, s_1\}$ corresponding to the solution $(10, 6)$. This solution remains optimal for all $\theta \ge 12$ since its reduced cost vector is

$$\overline{\mathbf{c}}_N = \begin{bmatrix} -\frac{13}{3} - \frac{5}{9}\theta & \frac{4}{3} - \frac{1}{9}\theta \end{bmatrix}.$$

Now consider $\theta = -\frac{8}{3}$, which generates the new optimal solution $(0, 15)$ with $\mathcal{B} = \{y, s_2, s_3\}$ and

$$\overline{\mathbf{c}}_N = \begin{bmatrix} 4 + \frac{3}{2}\theta & -5 - \frac{1}{2}\theta \end{bmatrix}.$$

This solution remains optimal for $-10 \le \theta \le -\frac{8}{3}$. When $\theta = -10$ we have $(0, 0)$ as our optimal solution with $\mathcal{B} = \{s_1, s_2, s_3\}$ and $\overline{\mathbf{c}}_N = (9 + 2\theta, 10 + \theta)$. This solution remains optimal for all $\theta \le -10$.

10.12 It is easiest to consider the dual of this problem

$$\begin{aligned}
\min \quad & (24 + \theta)y_1 + (24 + 2\theta)y_2 + (23 + 3\theta)y_3 \\
\text{s.t.} \quad & \\
& 4y_1 + y_2 + 3y_3 \ge 13 \\
& y_1 + 3y_2 + 2y_3 \ge 5 \\
& y_1, y_2, y_3 \ge 0
\end{aligned}$$

and use the parametric programming approach from Section 10.4. When $\theta = 0$, the optimal solution is $\mathbf{y} = (\frac{11}{5}, 0, \frac{7}{5})$ with dual values $\mathbf{x} = (5, 4)$ and basis $\mathcal{B} = \{y_1, y_3\}$. If we include $\theta(y_1 + 2y_2 + 3y_3)$ in the objective, this basic feasible solution has reduced cost

$$\overline{\mathbf{c}}_N = \begin{bmatrix} 7 - \frac{16}{5}\theta & 5 - \frac{1}{5}\theta & 4 + \frac{9}{5}\theta \end{bmatrix}.$$

Thus our current solution remains optimal for $-\frac{20}{9} \leq \theta \leq \frac{35}{16}$. When $\theta = \frac{35}{16}$ we have the new optimal basis $\mathcal{B} = \{y_1, y_2\}$ corresponding to the primal solution

$$\mathbf{x} = \left(\frac{48}{11} + \frac{1}{11}\theta, \frac{72}{11} + \frac{7}{11}\theta \right).$$

This primal basis remains optimal for $\theta \geq \frac{35}{16}$. Now consider $\theta = -\frac{20}{9}$. The new optimal basis is $\mathcal{B} = \{y_3, w_2\}$ with primal solution $\mathbf{x} = (\frac{23}{3} + \theta, 0)$. The reduced costs vector is

$$\overline{\mathbf{c}}_N = \begin{bmatrix} -\frac{20}{3} - 3\theta & \frac{49}{3} + \theta & \frac{23}{3} + \theta \end{bmatrix},$$

which remains nonnegative (and optimal) for $-\frac{23}{3} \leq \theta = -\frac{20}{9}$. If we set $\theta = -\frac{23}{3}$, we would have w_1 enter our basis and note that there is an unbounded direction in the dual, which indicates that for $\theta < -\frac{23}{3}$ there is no primal feasible solution.

Chapter 11

11.1 The optimal solution to the original problem has basis $\mathcal{B} = \{x, y, s_3\}$. After adding the new constraint, we have the basis $\mathcal{B} = \{x, y, s_3, s_4\}$ with basis matrix

$$B = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{bmatrix},$$

$\overline{\mathbf{c}}_N = (-1, -2)$ and $\mathbf{x}_B = (2, 6, 2, -1)$. Since $x_{s_4} < 0$, s_4 is our leaving variable, which generates

$$\mathbf{d}^T = \mathbf{e}_4^T B^{-1} N = \begin{bmatrix} -1 & -1 \end{bmatrix}.$$

Our ratio test gives

$$\begin{aligned} \lambda &= \min \left\{ \frac{\overline{c}_{s_1}}{d_{s_1}} = \frac{-1}{-1}, \frac{\overline{c}_{s_2}}{d_{s_2}} = \frac{-2}{-1} \right\} \\ &= 1, \end{aligned}$$

indicating s_1 is our entering variable. With the updated basis $\mathcal{B} = \{x, y, s_1, s_4\}$ we get $\overline{\mathbf{c}}_N = (-1, -2)$ and $\mathbf{x}_B = (1, 7, 1, 3)$, and hence our new solution $(x, y) = (1, 7)$ is optimal to the updated problem.

11.3 If we convert the constraints to equalities by adding slack/surplus variables, the initial basis $\mathcal{B} = \{s_1, s_2, s_3\}$ has $\overline{\mathbf{c}}_N = (-8, -3, -6, -5)$ and $\mathbf{x}_B = (-21, 30, -17)$, indicating an infeasible solution satisfying the optimality conditions. Since $x_{s_1} < 0$, s_1 is our leaving variable, generating

$$\mathbf{d}^T = \mathbf{e}_1^T B^{-1} N = \begin{bmatrix} -2 & 3 & -5 & -4 \end{bmatrix}.$$

Our ratio test gives

$$\begin{aligned} \lambda &= \min \left\{ \frac{\overline{c}_{x_1}}{d_{x_1}} = \frac{-8}{-2}, \frac{\overline{c}_{x_3}}{d_{x_3}} = \frac{-6}{-5}, \frac{\overline{c}_{x_4}}{d_{x_4}} = \frac{-5}{-4} \right\} \\ &= 6/5, \end{aligned}$$

indicating x_3 is our entering variable. With the updated basis $\mathcal{B} = \{x_3, s_2, s_3\}$ we get $\overline{\mathbf{c}}_N = (-\frac{28}{5}, -\frac{33}{5}, -\frac{1}{5}, -\frac{6}{5})$ and $\mathbf{x}_B = (\frac{21}{5}, \frac{24}{5}, -\frac{148}{5})$, which is still primal infeasible. We choose s_3 as our next leaving variable, which generates

$$\mathbf{d}^T = \mathbf{e}_3^T B^{-1} N = \begin{bmatrix} -\frac{26}{5} & -\frac{16}{5} & -\frac{22}{5} & \frac{3}{5} \end{bmatrix}.$$

Our ratio test gives x_4 is our entering variable. With the updated basis $\mathcal{B} = \{x_3, x_4, s_2\}$ we get $\bar{\mathbf{c}}_N = (-\frac{59}{11}, -\frac{71}{11}, -\frac{27}{22}, -\frac{1}{22})$ and $\mathbf{x}_B = (-\frac{13}{11}, \frac{74}{11}, \frac{556}{11})$, which is still primal infeasible. We choose x_3 as our next leaving variable, which generates

$$\mathbf{d}^T = \mathbf{e}_3^T B^{-1} N = \begin{bmatrix} -\frac{6}{11} & -\frac{13}{11} & -\frac{1}{11} & \frac{2}{11} \end{bmatrix}.$$

Our ratio test gives x_2 is our entering variable. With the updated basis $\mathcal{B} = \{x_2, x_4, s_2\}$ we get $\bar{\mathbf{c}}_N = (-\frac{31}{13}, -\frac{71}{13}, -\frac{19}{26}, -\frac{27}{26})$ and $\mathbf{x}_B = (1, 6, 40)$, which gives the optimal solution $(0, 1, 0, 6)$ to our original problem.

11.5 **Note: Errata here - supply s_1 should be 35 not 25. It is corrected here in the solutions.** Our current solution, the dual variables values (first fixing $u_1 = 0$), and the reduced costs (circled) are

$u_i \backslash v_j$	20	20	0	10	
0	$\boxed{20}$ 15	$\boxed{45}$ 25	$\boxed{35}$ 35	$\boxed{10}$ 20	35
15	$\boxed{35}$ 30	$\boxed{35}$ 20	$\boxed{50}$ 35	$\boxed{20}$ -5	50
15	$\boxed{30}$ -5	$\boxed{20}$ -15	$\boxed{15}$ 30	$\boxed{25}$ 10	40
	45	20	30	30	

.

We choose x_{32} are our entering variable, which generates the cycle $(3,2) \to (3,4) \to (1,4) \to (1,1) \to (2,1) \to (2,2) \to (3,2)$. Our leaving variable is x_{34} with $\theta = 10$, which generates the tableau

$u_i \backslash v_j$	20	20	15	10	
0	$\boxed{20}$ 5	$\boxed{45}$ 25	$\boxed{35}$ 20	$\boxed{10}$ 30	35
15	$\boxed{35}$ 40	$\boxed{35}$ 10	$\boxed{50}$ 20	$\boxed{20}$ -5	50
0	$\boxed{30}$ 10	$\boxed{20}$ 10	$\boxed{15}$ 30	$\boxed{25}$ 15	40
	45	20	30	30	

.

We choose x_{24} are our entering variable, which generates the cycle $(2,4) \to (1,4) \to (1,1) \to (2,1) \to (2,4)$. Our leaving variable is x_{14} with $\theta = 30$, which generates the tableau

$u_i \backslash v_j$	20	20	15	5	
0	$\boxed{20}$ 35	$\boxed{45}$ 25	$\boxed{35}$ 20	$\boxed{10}$ 5	35
15	$\boxed{35}$ 10	$\boxed{35}$ 10	$\boxed{50}$ 20	$\boxed{20}$ 30	50
0	$\boxed{30}$ 10	$\boxed{20}$ 10	$\boxed{15}$ 30	$\boxed{25}$ 20	40
	45	20	30	30	

.

Since all reduced costs are positive, this is an optimal solution.

11.11

(a) There are 33 possible patterns.

(b) Starting with the initial basis matrix

$$B = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

our simplex multipliers are $\mathbf{y} = \left(\frac{1}{5}, \frac{1}{3}, \frac{1}{2}, \frac{1}{2}, 1\right)$, we find that the pattern $\mathbf{a} = (0, 0, 1, 0, 1)$ has negative reduced cost and hence is an entering variable. Computing $\mathbf{d} = -B^{-1}\mathbf{a}$ and employing the ratio test, we find that the last column of B will leave.

In the next iteration, the simplex multipliers are $\mathbf{y} = \left(\frac{1}{5}, \frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$, we find that the pattern $\mathbf{a} = (1, 0, 2, 0, 0)$ has negative reduced cost and hence is an entering variable. Computing $\mathbf{d} = -B^{-1}\mathbf{a}$ and employing the ratio test, we find that the third column of B will leave.

In the next iteration, the simplex multipliers are $\mathbf{y} = \left(\frac{1}{5}, \frac{1}{3}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}\right)$, we find that the pattern $\mathbf{a} = (0, 0, 0, 1, 1)$ has negative reduced cost and hence is an entering variable. Computing $\mathbf{d} = -B^{-1}\mathbf{a}$ and employing the ratio test, we find that the fourth column of B will leave.

In the next iteration, the simplex multipliers are $\mathbf{y} = \left(\frac{1}{5}, \frac{1}{3}, \frac{2}{5}, \frac{2}{5}, \frac{2}{5}\right)$, we find that the pattern $\mathbf{a} = (0, 2, 1, 0, 0)$ has negative reduced cost and hence is an entering variable. Computing $\mathbf{d} = -B^{-1}\mathbf{a}$ and employing the ratio test, we find that the second column of B will leave.

In the next iteration, the simplex multipliers are $\mathbf{y} = \left(\frac{1}{5}, \frac{1}{3}, \frac{2}{5}, \frac{2}{5}, \frac{2}{5}\right)$, we find that all reduced costs are nonnegative and hence our current basic feasible solution $\mathbf{x}_B = (8, 48, 34, 100, 20)$ is optimal, using 210 patterns. The patterns are the columns of the matrix

$$B = \begin{bmatrix} 5 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

11.14 Adding slack variables $s_1, \ldots, s_5$ to the last 5 constraints gives the matrices

$$A_H = \begin{bmatrix} 2 & 1 & 3 & 2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad b_H = \begin{bmatrix} 18 \\ 10 \end{bmatrix}$$

corresponding to the hard constraints. If we start with the extreme points

$$\begin{aligned} \mathbf{v}_1 &= (0, 0, 0, 0, 7, 24, 10, 8, 4) \\ \mathbf{v}_2 &= (0, 7, 4, 0, 0, 3, 2, 0, 4) \\ \mathbf{v}_3 &= (6, 0, 4, 2, 1, 0, 0, 2, 0) \end{aligned}$$

for the easy constraints, we generate

$$B = \begin{bmatrix} 0 & 19 & 28 \\ 0 & 11 & 12 \\ 1 & 1 & 1 \end{bmatrix}, \alpha = \begin{bmatrix} 1/10 \\ 4/5 \\ 1/10 \end{bmatrix} \text{ and } \mathbf{c}_B = \begin{bmatrix} 0 \\ 63 \\ 54 \end{bmatrix}$$

for the master problem. This results in $\mathbf{y} = (\mathbf{w}, y_3) = (-\frac{81}{40}, \frac{369}{40}, 0)$ and the objective coefficients

$$\mathbf{c} - A_H^T \mathbf{w} = \left(-\frac{87}{40}, -\frac{11}{5}, \frac{77}{20}, -\frac{47}{40}, 0, 0, 0, 0, 0 \right).$$

Solving the linear program with these objective coefficients over the easy constraints yields the solution $\mathbf{v} = (0, 0, 4, 0, 7, 24, 2, 4, 0)$, giving the entering column $(A_H \mathbf{v}, 1) = (12, 4, 1)$ for the master problem. This gives $\mathbf{d} = \left(-\frac{7}{10}, \frac{2}{5}, -\frac{7}{10}\right)$. The ratio test (using α) gives either the first or third columns as leaving; we choose the first column, resulting in

$$B = \begin{bmatrix} 12 & 19 & 28 \\ 4 & 11 & 12 \\ 1 & 1 & 1 \end{bmatrix}, \alpha = \begin{bmatrix} 1/7 \\ 6/7 \\ 0 \end{bmatrix} \text{ and } \mathbf{c}_B = \begin{bmatrix} 28 \\ 63 \\ 54 \end{bmatrix}$$

for the master problem.

This results in $\mathbf{y} = (\mathbf{w}, y_3) = (-1, 6, 16$ and the objective coefficients

$$\mathbf{c} - A_H^T \mathbf{w} = (-1, 0, 4, 0, 0, 0, 0, 0).$$

The optimal solution to the resulting linear program is 0, so our current basis is optimal. Thus, the optimal extreme points to the easy problem are

$$\begin{aligned} \mathbf{v}_4 &= (0, 0, 4, 0, 7, 24, 2, 4, 0) \\ \mathbf{v}_2 &= (0, 7, 4, 0, 0, 3, 2, 4, 0) \\ \mathbf{v}_5 &= (0, 0, 4, 2, 7, 24, 0, 2, 0) \end{aligned}$$

which, along with $\alpha = (1/7, 6/7, 0)$ gives the actual optimal solution $(0, 6, 4, 0, 1, 6, 2, 4, 0)$ with value 58.

Chapter 12

12.1 The optimal distances from 1 are as follows:

i	1	2	3	4	5	6	7
$d(i)$	0	2	4	3	6	7	5

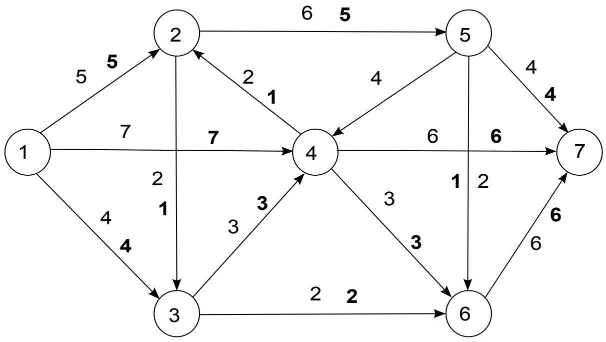
(a) For the FIFO label-correcting algorithm, we assume that the arcs are first ordered $A(1)$, $A(2), \ldots,$ $A(n)$. Initially we set $d(1) = 0$ and $List = \{1\}$. In the first pass we remove 1 from $List$ and examine $A(1)$. We update $d(2) = 2, d(4) = 3$ and have $List = \{2, 4\}$. In Pass 2 we first remove node 2 from $List$ and update $d(3) = 4$ and $d(5) = 6$. Next, we remove 4 from $List$ and update $d(6) = 7, d(7) = 5$, which results in $List = \{3, 5, 6, 7\}$. In Pass 3 we first remove 3, which updates no distances. Next we remove 5, then 6, and then 7, in each instance updating no distances. Our current distances are optimal.

(b) For Dijkstra's algorithm, we begin with $d(1) = 0$ and $List = \{1\}$. Removing 1 from $List$ and examining $A(1)$, we update $d(2) = 2$ and $d(4) = 3$, so that $List = \{2, 3\}$. Since 2 has the smallest distance label among those in $List$, it is removed and the arcs in $A(2)$ are examined. This updates $d(3) = 4$ and $d(5) = 6$, making $List = \{3, 4, 6\}$. We next remove 4 from $List$ and update $d(6) = 7$, $d(7) = 5$. In the next iteration we remove 3 from $List$, but no updates occur. Subsequent iterations remove (in order) $7, 5, 6$, and in each case no updates occur. Our current distances are optimal.

12.7 The given paths were identified on the residual network in the following order:

Path	Value Change δ
$1 \to 2 \to 5 \to 7$	$\delta = 4$
$1 \to 2 \to 5 \to 6 \to 7$	$\delta = 1$
$1 \to 3 \to 4 \to 7$	$\delta = 3$
$1 \to 3 \to 6 \to 7$	$\delta = 1$
$1 \to 4 \to 7$	$\delta = 3$
$1 \to 4 \to 6 \to 7$	$\delta = 3$
$1 \to 4 \to 2 \to 3 \to 6 \to 7$	$\delta = 1$

The resulting flow given in the figure below (with flow values in bold) has value $v^* = 16$. An optimal s-t cut is $S = \{1\}, \overline{S} = \{2, 3, 4, 5, 6, 7\}$ with $u[S, \overline{S}] = 16$.



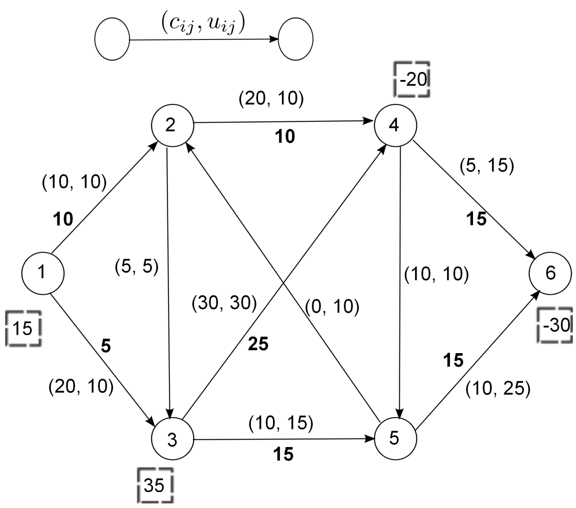
12.13 Our initial basic feasible solution has $\mathcal{T} = \{(1,2), (2,3), (5,2), (4,6), (5,6)\}$ and $\mathcal{U} = \{(1,3), (2,4), (3,4), (3,5), (4,5)\}$. If we set $y_2 = 0$ and solve for the other dual variables using the arcs in $\mathcal{T}$, we get $y_1 = 10$, $y_3 = -5$, $y_4 = -5$, $y_5 = 0$, $y_6 = -10$, which results in the reduced costs $\bar{c}_{13} = 5$, $\bar{c}_{24} = 15$, $\bar{c}_{34} = 30$, $\bar{c}_{35} = 15$, $\bar{c}_{45} = 15$; all are candidates to enter the basis. If we choose arc $(3,4)$ to enter, we find that $(2,3)$ would leave $(\theta = 0)$, resulting in $\mathcal{T} = \{(1,2), (3,4), (5,2), (4,6), (5,6)\}$, $\mathcal{L} = \{(2,3)\}$, and $\mathcal{U} = \{(1,3), (2,4), (3,5), (4,5)\}$.

If we set $y_2 = 0$ and solve for the other dual variables using the arcs in $\mathcal{T}$, we get $y_1 = 10, y_3 = 25, y_4 = -5, y_5 = 0, y_6 = -10$, which results in the reduced costs $\bar{c}_{13} = 35$, $\bar{c}_{23} = 30$, $\bar{c}_{24} = 55$, $\bar{c}_{35} = -15$, $\bar{c}_{45} = 15$. If we choose arc $(1,3)$ to enter, we find that $(1,2)$ would leave $(\theta = 5)$, resulting in $\mathcal{T} = \{(1,3), (3,4), (5,2), (4,6), (5,6)\}$, $\mathcal{L} = \{(2,3)\}$, and $\mathcal{U} = \{(1,2), (2,4), (3,5), (4,5)\}$.

If we set $y_2 = 0$ and solve for the other dual variables using the arcs in $\mathcal{T}$, we get $y_1 = 45, y_3 = 25, y_4 = -5, y_5 = 0, y_6 = -10$, which results in the reduced costs $\bar{c}_{12} < 0$, $\bar{c}_{23} > 0$, $\bar{c}_{24} = 15$, $\bar{c}_{35} < 0$, $\bar{c}_{45} = 15$. If we choose arc $(4,5)$ to enter, we find that $(4,6)$ would leave $(\theta = 10)$, resulting in $\mathcal{T} = \{(1,3), (3,4), (5,2), (4,5), (5,6)\}$, $\mathcal{L} = \{(2,3)\}$, and $\mathcal{U} = \{(1,2), (2,4), (3,5), (4,6)\}$.

If we set $y_2 = 0$ and solve for the other dual variables using the arcs in $\mathcal{T}$, we get $y_1 = 60, y_3 = 40, y_4 = 10, y_5 = 0, y_6 = -10$, which results in the reduced costs $\bar{c}_{12} < 0$, $\bar{c}_{23} > 0$, $\bar{c}_{24} = 30$, $\bar{c}_{35} < 0$, $\bar{c}_{46} < 0$. If we choose arc $(2,4)$ to enter, we find that $(5,2)$ would leave $(\theta = 10)$, resulting in $\mathcal{T} = \{(1,3), (3,4), (2,4), (4,5), (5,6)\}$, $\mathcal{L} = \{(2,3), (5,2)\}$, and $\mathcal{U} = \{(1,2), (3,5), (4,6)\}$.

If we set $y_2 = 0$ and solve for the other dual variables using the arcs in $\mathcal{T}$, we get $y_1 = 30, y_3 = 10, y_4 = -20, y_5 = -30, y_6 = -40$, which results in the reduced costs $\bar{c}_{12} < 0$, $\bar{c}_{23} > 0$, $\bar{c}_{52} >=$, $\bar{c}_{35} < 0$, $\bar{c}_{46} < 0$. Thus our current solution is optimal, and is in the figure below.



13.15 **Errata: should have initial solution with** $x_{Det,Den} = 60$ **and** $x_{Det,Cin} = 50$.

Chapter 13

13.1 One possible formulation is

$$\widehat{X} = \left\{ \mathbf{x} \in \{0,1\}^5 : \begin{array}{r} 3x_1 + 2x_2 + 5x_3 + 4x_4 + 2x_5 \leq 13 \\ x_1 + x_2 + x_3 + x_4 \leq 3 \end{array} \right\}.$$

Note that $\widehat{X} \subset X$ by definition and because we have $(1,1,1,0.75) \in X - \widehat{X}$.

13.3 (**Errata:** constraint $3x + 2y \leq 3$ should be $3x + 2y \geq 3$) The ideal formulations is

$$P = \left\{ (x,y) : \begin{array}{rcl} x + y & \geq & 1 \\ x - y & \geq & 1 \\ y & \leq & 2 \\ x & \leq & 2 \\ x, y & \geq & 0 \end{array} \right\}.$$

13.11 **Errata:** The matrix A should be

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

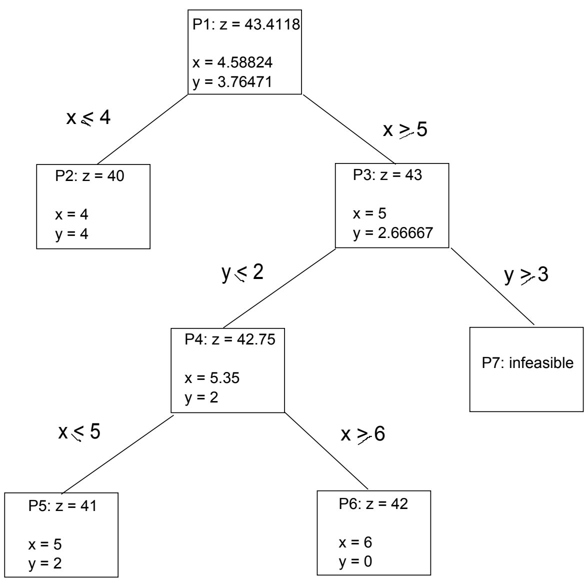
13.12 **Errata:** The matrix A should be (after negating the second row)

$$A = \begin{bmatrix} 1 & 0 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 \end{bmatrix}$$

Chapter 14

14.1 (**Errata:** The right-hand side of the second constraint should be 48 instead of 45)

14.2 Solving the root problem $P1$ gives the solution $x = 4.58824$, $y = 3.76471$ with value $z = 43.4118$. If we branch on x by first generating the subproblem $P2$ where the $x \leq 4$ is added, we get the optimal solution $x = 4$, $y = 4$ with $z = 40$. Since this is an integer solution we use this as our incumbent. Next we solve problem $P3$ where the constraint $x \geq 5$ is added to problem $P1$; this gives the solution $x = 5$,$y = 2.666667$ and $z = 43$. Our next subproblem $P4$ is created by adding the constraint $y \leq 2$ to $P3$ (branching on y), which yields the solution $x = 5.35$, $y = 2$, with $z = 42.75$. Next, we branch on x and problem $P5$ is generated from $P4$ by adding the constraint $x \leq 5$, giving the integer solution $x = 5, y = 2$ with $z = 41$, which is our new incumbent solution. We the generate problem $P6$ from $P4$ by adding the constraint $x \geq 6$, which generates the integer solution $x = 6, y = 0$ with $z = 42$, which is our new incumbent. Next, we return to problem $P3$ and branch on y using the constraint $y \geq 3$ to create problem $P7$; this problem is infeasible. We now have no more problems to branch from, and so our incumbent solution $(6, 0)$ is optimal. The branch and bound tree is given below.



14.5 (**Errata:** All constraints should be "$\geq$" instead of "$\leq$")

14.11 None of the inequalities are valid, as each is violated by $\left(\frac{78}{17}, \frac{64}{17}\right)$.

14.12 If we add slack variables s_1 and s_2 to the constraints, a basic feasible solution corresponding to $(2.5, 0)$ is $(2.5, 0, 19, 0)$ with basis $\mathcal{B} = \{x, s_1\}$. We then have

$$B = \begin{bmatrix} -4 & 1 \\ 10 & 0 \end{bmatrix} \qquad \text{and} \qquad B^{-1} = \begin{bmatrix} 0 & 1/10 \\ 1 & -2/5 \end{bmatrix}.$$

Solving for the basic variables in terms of the nonbasic variables yields

$$\begin{aligned} x &= 2.5 - \frac{2}{5}y - \frac{2}{5}s_2 \\ s_1 &= 19 - \frac{1}{10}y + \frac{2}{5}s_2. \end{aligned}$$

Using the rounding procedure with the first equation, we get the C-G valid inequality $x \leq 2$ which is violated by $(2.5, 0)$.

14.14 The initial LP-relaxation solution has $x = \frac{5}{3}$, $y = \frac{10}{3}$. Adding slack variables to each equation and solving for the basic variables x, y we get

$$\begin{aligned} x &= \frac{5}{3} - \frac{1}{3}s_1 + \frac{7}{3}s_2 \\ y &= \frac{10}{3} + \frac{1}{3}s_1 - \frac{10}{3}s_2. \end{aligned}$$

These generate the violated C-G cutting planes $x - 3s_2 \leq 1$ and $y - s_1 + 3s_2 \leq 3$. Adding these to our LP-relaxation yields the new solution $(x, y) = (4, 0)$, which is the optimal integer solution.

14.17 The minimum covers and their cover inequalities are as follows:

$\{1,2,4\}$	$x_1 + x_2 + x_4 \leq 2$
$\{1,3,5\}$	$x_1 + x_3 + x_5 \leq 2$
$\{1,4,5\}$	$x_1 + x_4 + x_5 \leq 2$
$\{2,3,4\}$	$x_2 + x_3 + x_4 \leq 2$
$\{2,5\}$	$x_2 + x_5 \leq 1$
$\{3,4,5\}$	$x_3 + x_4 + x_5 \leq 2$

14.21 (**Errata: In the problem statement, the coefficient of x_1 should be 10 and the coefficient of x_6 should be 1**)

14.21(b) (**Errata: The solution should be $\mathbf{x} = (1, \frac{2}{3}, \frac{1}{3}, 0, \frac{2}{3}, \frac{1}{3})$.**)

14.25 (**Errata:** The right-hand side value in the constraint defining S should be 18 and not 17.)

Chapter 15

Because many of the exercises ask for an algorithm to be designed and implemented on a particular problem, no solutions are given in many cases. There can be multiple such algorithms, depending on the search criteria used.

15.9 (a) The offspring would be $O_1 = (1, 3, 4, 5, 2, 5, 8, 7)$ and $O_2 = (4, 2, 6, 5, 1, 3, 8, 7)$

15.10 **Errata:** P_2 **should be** $(1, 5, 2, 3, 6, 4)$**.**