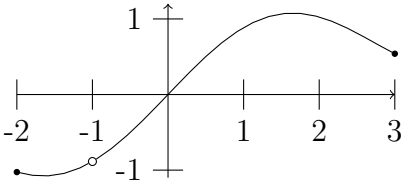
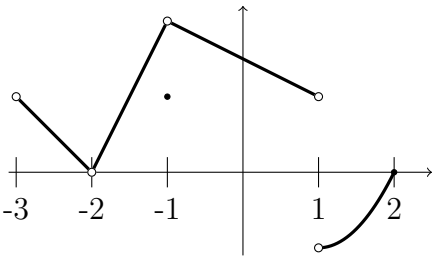
MA111 - Homework Sec. 2.5

You may work together but do not copy other peoples' work and present it as your own.

1 Where on $[-2, 3]$ does the function graphed here fail to be continuous and why is it not continuous there?



2 For the graph shown here



a. Does $f(-1)$ exist?
b. Does $\lim_{x\to -1} f(x)$ Exist?
c. Does $\lim_{x\to -1} f(x) = f(-1)$?
d. If f continuous at $x = -1$?

At what points are the functions in exercises **3-4** continuous?

3 $y = \dfrac{1}{x-3} - 5x$

4 $y = \dfrac{x\cot(x)}{x^2+1}$

5

a. Find the limit $\lim_{x\to\pi} \cos(x - \sin(x))$
b. Is $\cos(x - \sin(x))$ continuous at $x = \pi$?

6 Define $g(4)$ is a way that extends $g(x) = \dfrac{x^2-16}{x-4}$ to be continuous at $x = 4$.

7 Graph the function near $x = 0$. Zoom as needed to find a good candidate for the extended function's value at $x = 0$. If the function does not appear to have a continuous extension, can it be extended to be continuous from the right or from the left? If so, what do you thisnk the extended functions value(s) should be?

$f(x) = \dfrac{20^x - 1}{x}$

8 Show that the equation $x^3 - 15x + 1 = 0$ has three solutions in the interval $[-4, 4]$ be finding positive and negative values of $f(x)$ an using the intermediate value theorem.

9 Is it true that if you stretch a rubber band by moving one end to the right and the other to the left some point in the band will end up in its original position? Give reasons for your answer.

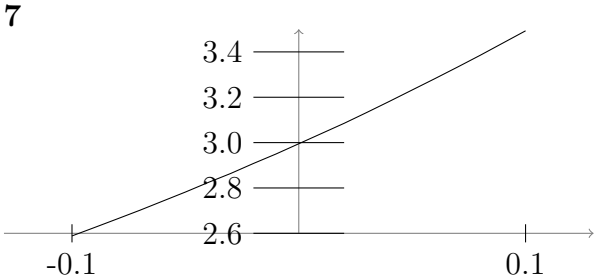
Some answers and suggestions.

1 $x = -1$. $f(-1)$ does not exist.

3 $x = 3$. The function is discontinuous when the denominator is zero. This happens at $x - 3 = 0$.

5 Because sine and cosine are continuous everywhere we can plug in $x = \pi$. $\cos(\pi - \sin(\pi)) = \cos(\pi - 0) = -1$.

7



Zoom in further to get a more accurate value.

9 Let x represent the starting position of points on the rubber band. Let $f(x)$ represent the final position of point x. Look at $g(x) = x - f(x)$ for the smallest and largest values of x on the rubber band.